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Fuzzy risk matrix

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ABSTRACT

A risk matrix is a mechanism to characterize and rank process risks that are typically identified through one or more multifunctional reviews (e.g., process hazard analysis, audits, or incident investigation). This paper describes a procedure for developing a fuzzy risk matrix that may be used for emerging fuzzy logic applications in different safety analyses (e.g., LOPA). The fuzzification of frequency and severity of the consequences of the incident scenario are described which are basic inputs for fuzzy risk matrix. Subsequently using different design of risk matrix, fuzzy rules are established enabling the development of fuzzy risk matrices. Three types of fuzzy risk matrix have been developed (low-cost, standard, and high-cost), and using a distillation column case study, the effect of the design on final defuzzified risk index is demonstrated.

1. Introduction

Prevention of accidents and other unwanted events possibly occurring during industrial process activities handling of hazardous materials requires a risk assessment. During that exercise the two components of risk are evaluated:

- the occurrence of undesirable consequences, and
- the likelihood of occurrence of this consequence.

There are mainly three ways to perform a risk assessment:

1. qualitative way, 2. semi quantitative way, and 3. quantitative way.

The first method of evaluation is based only on the compliance assessment, the second method applies to the categorization of those components and final risk score is achieved using different methods, and the third one is based on risk measures assessment within the QRA.

In this paper the semi quantitative approach is taken into account and risk matrix is used for the risk evaluation and assessment. In traditional approach the risk factors are expressed by crisp categories, and number of categories for each risk factor depends on the analytics.

However, in process risk analysis, real situation is often not crisp and deterministic due to number of uncertainties. The latter may be classified into two groups: as "objective uncertainties" which arise from a random character of the assessment process (variability), and "subjective uncertainties", arising from limited and partial knowledge and information (imprecision). In such a situation a fuzzy logic can be used. According to Zadeh [1] fuzzy logic or fuzzy set theory can work with uncertainty and imprecision and can solve problems where there are no sharp boundaries and precise values. The concept of a fuzzy set provides mathematical formulations that can characterize the uncertain parameters involved in particular risk analysis method. In such a way all risk components were expressed in terms of fuzzy sets and similarly to traditional risk matrix (TRM) the fuzzy risk matrix was developed. That may be subsequently used for other PHA analysis methods including LOPA [2].

2. Traditional risk assessment matrix approach (TRM)

Risk assessment matrix is a tool to conduct subjective risk assessment for use in different process hazard analysis (PHA), including the layer of protection analysis (LOPA). The bases for risk matrix are the definition of risk as a combination of severity of the consequences occurring in a certain accident scenario and its frequency.

In order to build risk matrix the following steps need to be undertaken:

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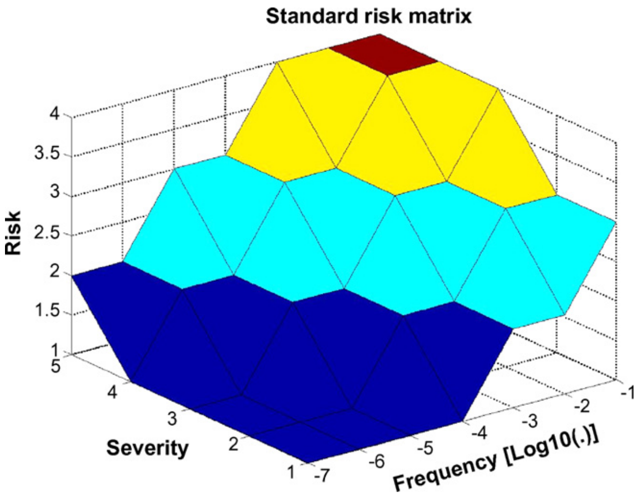


Fig. 1. Standard risk matrix surface.

1. categorization and scaling of the severity of consequences and frequency,
2. categorization and scaling of output risk index,
3. build-up risk-based rules knowledge,
4. graphical edition of the risk matrix.

The categorization of the severity and frequency depends on the type of activity or specifics of the processes involved. In general, according to standard MIL-STD-882D [3] a frequency is categorized into six categories and severity of the consequences into four categories. This is a basis to constitute the plane matrix with 24 cells each representing a certain risk category. Sometimes, especially for simple risk assessments, there may be used 3 × 3 cells matrix, 5 × 5 [4], and for process plants risk assessment of the larger structure, like 7 × 4 [5]. This work advocates 7 × 5 cells risk matrix—meaning that there are 7 different levels of probabilities and 5 different levels of severity of consequences. This matrix has 35 risk cells.

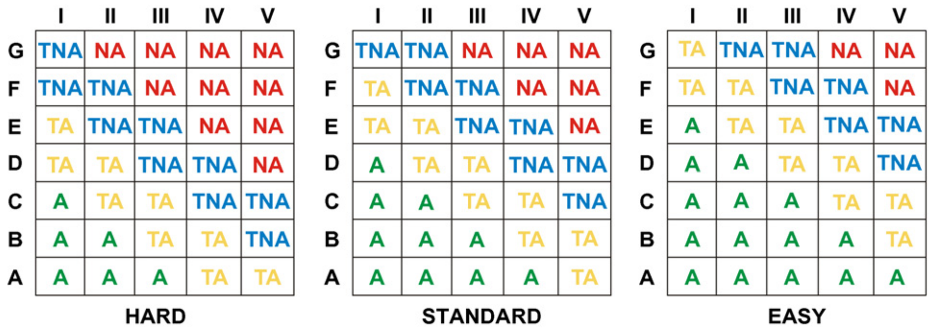
The relationships between all input and output for standard risk matrix are shown graphically in Fig. 1.

In the next step the risk categorization takes place. In general three or four risk categories are selected. In that work we have applied four risk categories: A: acceptable, no further action is required; TA: tolerable acceptable, further action is based on ALARP principle; TNA: tolerable–unacceptable, additional safety measures are required; and NA: non-acceptable, must change immediately.

The relation between frequency, severity and risk categories is described by the risk-based engineering rules. This is presented by the classical logic implication as follows:

IF frequency is “f” category AND severity of consequences is “c” category THEN risk is “r” category.

The above risk rules are obvious for the boundary categories of the frequency and severity, e.g. IF frequency is “unlikely B” and severity of consequence is “negligible I” THEN the risk category may be assessed as an “acceptable A” only. The situation is more difficult for intermediate categories of severity and frequency. In such cases an expert opinion is applied with the application of an interpolation



	I	II	III	IV	V
G	TNA	NA	NA	NA	NA
F	TNA	TNA	NA	NA	NA
E	TA	TNA	TNA	NA	NA
D	TA	TA	TNA	TNA	NA
C	A	TA	TA	TNA	TNA
B	A	A	TA	TA	TNA
A	A	A	A	TA	TA

HARD

	I	II	III	IV	V
G	TNA	TNA	NA	NA	NA
F	TA	TNA	TNA	NA	NA
E	TA	TA	TNA	TNA	NA
D	A	TA	TA	TNA	TNA
C	A	A	TA	TA	TNA
B	A	A	A	TA	TA
A	A	A	A	A	TA

STANDARD

	I	II	III	IV	V
G	TA	TNA	TNA	NA	NA
F	TA	TA	TNA	TNA	NA
E	A	TA	TA	TNA	TNA
D	A	A	TA	TA	TNA
C	A	A	A	TA	TA
B	A	A	A	A	TA
A	A	A	A	A	A

EASY

Fig. 2. Risk assessment matrices (frequency categories: A: remote, B: unlikely, C: very low, L: low, M: medium, H: high, G: very high; severity categories: I: negligible, II: low, III: moderate, IV: high, V: catastrophic; risk categories: A: acceptable, TA: tolerable–acceptable, TNA: tolerable–unacceptable, NA: unacceptable).

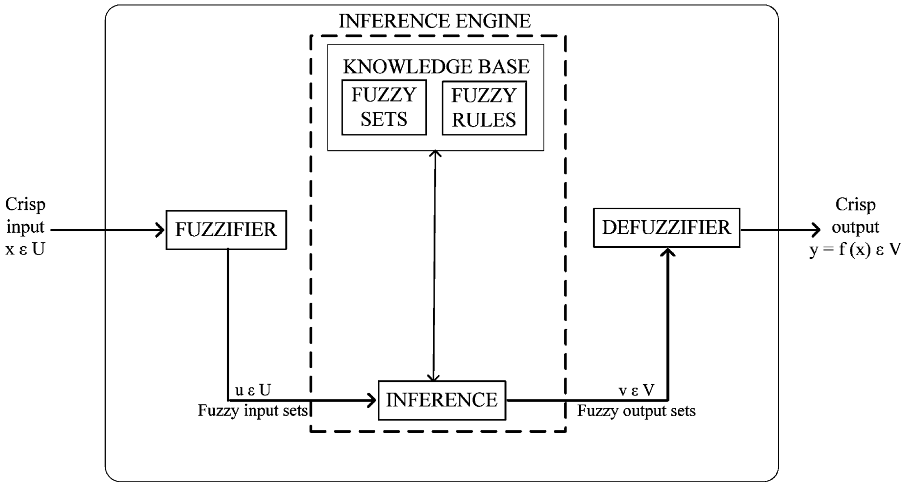


Fig. 3. The structure of a typical FLS.

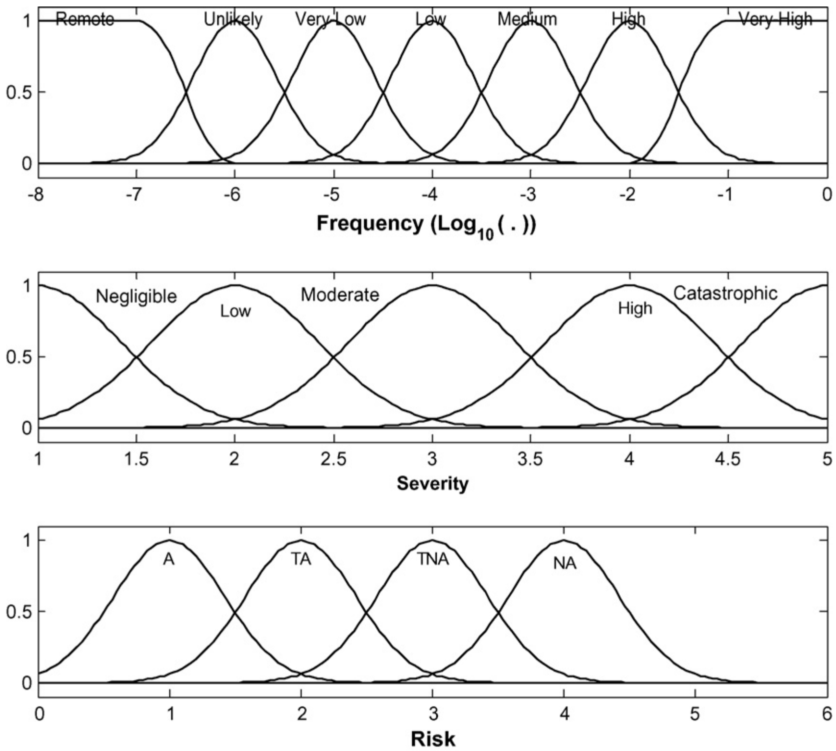


Fig. 4. Membership functions for fuzzy risk assessment.

scheme. Of course such an opinion may be quite subjective and imprecise.

The categorization of all parameters and the risk rules provide a risk tolerance zoning and constitutes the risk matrix. In order to alter risk tolerance limits two methods are applied: 1. rezoning of cells within the matrix, and 2. redefining severity or frequency categories. Fig. 2 presents three different risk matrixes, with rezoning, used in this research. The "standard" matrix represents a typical risk matrix encountered in many process industries. The "hard" matrix represents the high cost matrix however safer, whereas the "easy" matrix provides lower categories of risk, which may imply less layers of protection are needed to achieve safety assurance. Of course it will definitely increase the potential for accidents and incidents. It means that it is a low-cost matrix, however less safe. These matrices recognize different risk tolerance limits and they may represent different safety assessment strategy.

The application of risk matrix is simple. After assessment of the severity and frequency categories the risk category as one out of four categories (A, TA, TNA, NA) is specified using risk matrix. This is a basis for further risk control measures. Note that procedures, which use qualitative verbal descriptors, e.g. low, high, or possible, are quite vague and imprecise, however risk analysts frequently use them. Uses of such value judgments introduce uncertainty that is a result of fuzziness, not randomness [6].

The selection of proper risk matrix is an important management task that is included into safety policy or safety program, e.g. for SEVESO industry it may be included into major accident prevention policy [7].

TRM has got some advantages and disadvantages. The most important is that it provides a standard tool for treating the relationship between the severity of consequences and the probability in assessing process risks and main disadvantages are connected

Table 1
Fuzzy sets for fuzzy risk matrix

Linguistic variables of risk factors	Linguistic term (fuzzy set)	Description range	Universe of discourse (*X*)
Frequency (F)	Very high-category G	G ($1 < F > 10^{-2}$) [1/year]	$X_F \in (10^0, 10^{-8})$
	High: F	F ($10^{-1} \leq F < 10^{-3}$)	
	Moderate: E	E:($10^{-2} \leq F < 10^{-4}$)	
	Low: D	D ($10^{-3} \leq F < 10^{-5}$)	
	Very low: C	C ($10^{-4} \leq F < 10^{-6}$)	
	Unlikely: B	B ($10^{-5} \leq F < 10^{-7}$)	
	Remote: A	A ($F < 10^{-6}$)	
Severity of consequences (C)	Negligible (no losses)	$1 < C \leq 2$	$X_C \in (1, 5)$
	Low (lost day work)	$2 < C \leq 3$	
	Moderate (injury)	$3 < C \leq 4$	
	High (disabilities)	$3 \leq C \leq 5$	
	Catastrophic (fatalities)	$C > 5$	
Risk category (R)	A: acceptable—no action required	$0 < R \leq 2$	$X_R \in (0, 5)$
	TA: tolerable–acceptable (action based on ALARP principle)	$1 \leq R \leq 3$	
	TNA: tolerable–unacceptable (indication for improvements in medium notice	$2 \leq R \leq 4$	
	NA: unacceptable (must reduce immediately)	$3 \leq R \leq 5$	

with large uncertainties and that it cannot assist in identifying the hazards.

3. Fuzzy risk matrix development (FRM)

In order to overcome uncertainties and imprecision connected with the TRM the fuzzy logic (FL) was employed. FL can work with uncertainty and imprecision and can solve problems where there are no sharp boundaries and precise values [1]. Such a situation is in the concept of risk assessment matrix. In fuzzy logic, the equivalent to traditional independent variables, fuzzy sets are defined for specific linguistic variables, i.e. frequency, severity of the consequences and risk. The selected categories of each variable constitute the fuzzy sets. A fuzzy set defined on a universe of discourse (U) is characterized by a membership function, $\mu(x)$, which takes on values from the interval [0, 1]. A membership function provides a measure of the degree of similarity of an element in U to the fuzzy subset.

Fuzzy risk matrix development requires an application of the fuzzy logic system (FLS), which is shown in Fig. 3 [8].

The FLS consists of the following elements:

1. The fuzzifier maps crisp input into fuzzy sets. It means that during fuzzification for each risk matrix component (frequency, severity and risk) appropriate fuzzy sets are formed according to fuzzy set principles using knowledge base.
2. The inference engine of the FLS maps input fuzzy sets, by means of a set of rules, into fuzzy output sets. In handles the way in which rules are combined. These set of rules are generated from engineering knowledge by means of the collection of IF-THEN statements. It allows for fuzzy risk assessment.
3. Defuzzification is the process of weighting and averaging the outputs from all of the individual fuzzy rules into one single output value. This output decision, concerning risk index is a precise, defuzzified, and has crisp value.

3.1. Fuzzy risk matrix sets definition (fuzzification)

To develop fuzzy risk assessment matrix, relevant and available input variables must be selected and their domain is partitioned in a number of fuzzy sets. TRM provides data for the number of sets as well as for their range. Table 1 gives the details of fuzzy sets applied in the fuzzification step.

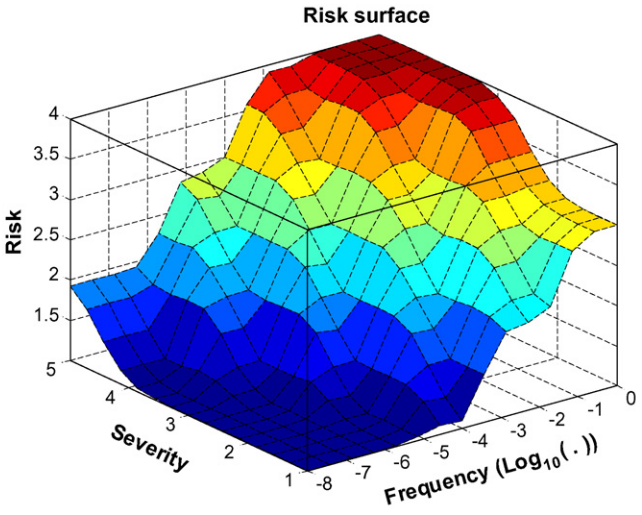


Fig. 5. Fuzzy risk surface (standard).

Table 2
A comparison of risk assessment results

Accident scenario	Input				Output					
	Crisp		Fuzzy		Traditional risk matrix R			Fuzzy risk matrix $\bar{R}$		
	F	S	$\bar{F}$	$\bar{S}$	Standard	Easy	Hard	Standard	Easy	Hard
RAS(R)1	1E−7	5	1.44E−7	4.35	2 (TA)	1 (A)	2 (TA)	1.35 (TA-0.75, A-0.25)	1.18 (TA-0.1, A-0.9)	2.18 (TA-0.85, A-0.15)
RAS(L)2	5E−6	5	6.27E−6	4.08	2 (TA)	2 (TA)	3 (TNA)	2.01 (TA-1.0)	1.84 (TNA-0.01, A-0.014, TA-0.85)	2.97 (TNA-0.98, NA-0.01, TA-0.01)
RAS(R)3	1E−7	5	1.44E−7	4.55	2 (TA)	1 (A)	2 (TA)	1.56 (TA-0.48, A-0.52)	1.23 (A-0.92, TA-0.08)	2.56 (TNA-0.51, TA-0.49)
RAS(L)4	SE−6	5	6.27E−6	4.28	2 (TA)	2 (TA)	3 (TNA)	2.25 (TA-0.35, A-0.65)	1.83 (TA-0.12, TNA-0.02, TA-0.86-TA)	3.01 (TNA-0.98, TA-0.01, NA-0.01)

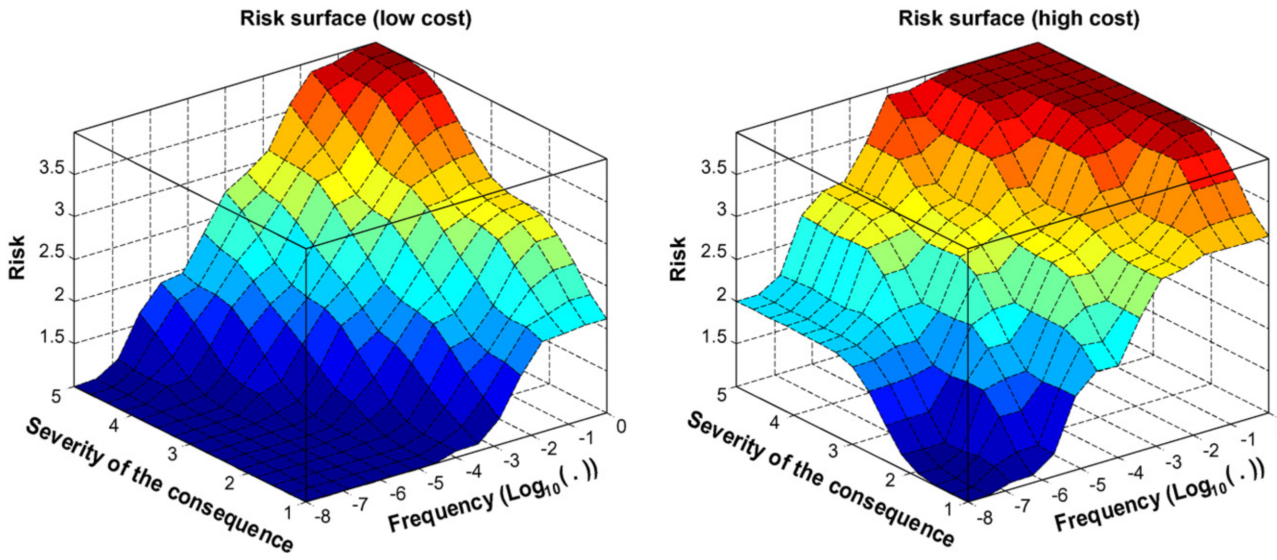


Fig. 6. Two variants of the risk matrix (EASY-left), (HARD-right).

The ranges of frequencies and severities of the consequences were reconverted from the look-up table provided by LOPA book [9].

Different forms of a membership function can be used depending on the type of the characteristics of input and output variables. In this research the Gaussian type of membership function was selected as the most natural and popular choice for these systems. Fig. 4 presents the fuzzy sets and its membership function for each variable used in the fuzzy risk assessment matrix.

3.2. *Fuzzy inference system*

A fuzzy inference system applies risk rules-based knowledge in mapping of fuzzy input sets (frequency and severity) into fuzzy output risk set. It is performed by fuzzy IF-THEN rules. The structure of fuzzy rules for the fuzzy risk matrix can be presented as follows:

IF frequency is $\bar{f}_n$ AND severity of consequences is $\bar{c}_m$ THEN risk is $\bar{r}_z$, where $\bar{f}_n$, $\bar{s}_m$, $\bar{r}_z$ are the fuzzy sets for frequency $\bar{F}$, severity $\bar{S}$ and risk $\bar{R}$ defined on the universes of discourse, respectively.

Fuzzy rules are provided by TRM. A combination of 7 categories of frequency and 5 categories of severity (called antecedents), according to the assumed structure of risk matrix, generates 35 rules providing 35 conclusions, which represent risk categories. In order to transfer the qualitative rules into quantitative result a Mamdani fuzzy inference algorithm is applied [8]. The Mamdani model applies min operator for AND method and implication of the output set. After the rules have been evaluated, the output fuzzy set for each rule was aggregated. The aggregating output membership function of a resultant output fuzzy risk category is expressed as

$$\mu_{\bar{R}}(r) = \max_{k}\{\min \mu^{k}_{\bar{F}}(f_n), \mu^{k}_{S}, \mu^{k}_{\bar{R}}(r_z)\}$$

where k is the number of rules, n the number of fuzzy frequency sets, m the number of fuzzy severity sets, and z is the number of fuzzy risk sets.

3.3. *Defuzzification*

The conversion of final combined fuzzy conclusion into a crisp (nonfuzzy) form is called the defuzzification. There are numbers of available defuzzification techniques [8]. In this work we have applied the center of area (COA) or the centroid method. The COA calculates the weighted average of a fuzzy set. The result of applying COA defuzzification for risk index can be expressed by the formula:

$$r = \frac{\int \mu_{\bar{R}}(r) r \, \mathrm{d}r}{\int \mu_{\bar{R}}(r) \, \mathrm{d}r}$$

3.4. *Fuzzy risk surface*

The relationship between frequency, severity and risk can be illustrated by three-dimensional plot that represents the mapping from two inputs (frequency and severity) to one output (risk). This is a risk surface. Fig. 5 shows such a surface for "standard" risk matrix and Fig. 6 illustrates risk surface for "Easy" and "Hard" risk matrix.

The risk surfaces present different the regions of risk depending on input parameters and can be used for risk assessment. The above plots present the differences in each proposed risk matrix. The characteristic mean risk index for standard matrix is 2.18, for "easy" matrix is 1.91 and for "hard" matrix is equal 2.75.

4. Case study

As an example the distillation column unit presented in detail in [2] was selected. Four accident scenarios were identified representing rupture of a column due to overpressure caused by loss of cooling RAS(R), and leak from relieve valve (RV) due to high pressure caused by failure of cooling, RAS(L).

A comparison of the results using different risk matrixes (TRM and FRM) with different risk zoning design is presented in Table 2.

Comparing the crisp data on risk category received by the TRM and fuzzy risk indexes by the FRM it can be stated that the results are more precise and describe in detail the possible contribution of each fuzzy set in a final result. This may be of help in a more accurate design of the risk control measures or the layers of protection.

A complete evaluation procedure involving all rules for the run RAS(R)1 according to the Mamdani model is illustrated in Fig. 7. From this we see that the fuzzy risk index equal 1.35 (for a standard risk matrix) is contained in the fuzzy sets TA and A with memberships 0.85 and 0.15, respectively.

Table 2 gives the comparison of the standard fuzzy risk index $\bar{R}$, with these obtained from Easy (low-cost) and Hard (higher cost)

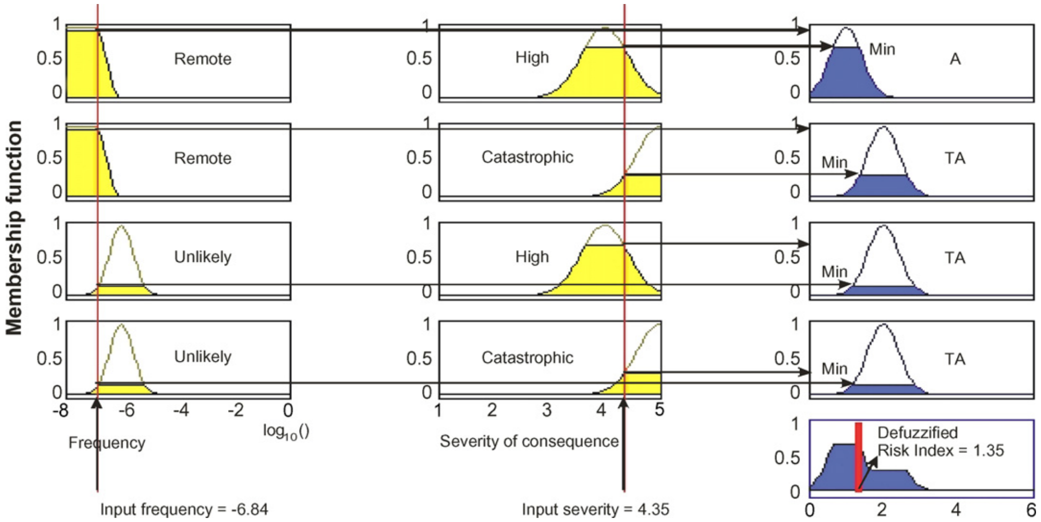


Fig. 7. Rules inference process for the RAS(R) case.

risk matrices. The risk index increases with the level of riskiness, which means that in order to meet at least risk tolerance criteria in the case of hard matrix we will need to spend more (but with a better protection) than in the case of the easy matrix. An opposite situation will result in the benefits due to reduction of accidents and other losses. The above means that the final risk score strongly depends on the structure of the risk assessment matrix.

5. Conclusions

1. Risk matrix is a very useful tool for semi-quantitative risk assessment as well as a selection of risk control measures.
2. The overall risk category (risk score) obtained in a traditional approach by categorization of frequency and severity of the consequence is quite imprecise and vague which produces the significant uncertainties concerning the risk category.
3. One of the methods to deal with such uncertainties in risk assessment is a fuzzy logic where fuzzy sets are a fundamental issue. In contrast to the TRM, all variables of the risk matrix are expressed in fuzzy sets defined by appropriate membership functions. An application of the FLS allow for mapping input data into output results
4. The data of case studies indicate that the final risk result obtained due to the fuzzy risk matrix application is more precise and reliable.
5. An effect of different risk surface can be used for the design of a more reliable safety system assurance.

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